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RECENT DEVELOPMENTS OF SPH IN MODELING EXPLOSION AND IMPACT PROBLEMS

M. B. LIU[1], D. L. FENG[1] AND Z. M. GUO[2,3]

[1] Institute of Mechanics
Chinese Academy of Sciences
Beijing 100190, China
e-mail: liumoubin@imech.ac.cn, fengdianlei@imech.ac.cn

[2] Laboratori de Càlcul Numèric (LaCàN)
Universitat Politecnica de Catalunya (UPC)
C. Jordi Girona 1-3, Campus Nord, 08034 Barcelona, Spain
e-mail: guozhiming@live.cn

[3] North University of China
Xueyuan Road 3
030051 Taiyuan, Shanxi, China
e-mail:guozhiming@live.cn

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Abstract. Explosion and impact problems are generally characterized by the presence of shock waves, intense localized materials response and intensive loadings. Most of the wave propagation hydro-codes for such problems use traditional grid based methods such as finite difference methods (FDM) and finite element methods (FEM). Though many successful achievements have been made using these methods, some numerical difficulties still exist. These numerical difficulties generally arise from large deformations, large inhomogeneities, and moving interfaces, free or movable boundaries. Smoothed particle hydrodynamics (SPH) is a Lagrangian, meshfree particle method, and has been widely applied to different areas in engineering and science. SPH method has been intensively used for simulating high strain hydrodynamics with material strength, due to its special features of meshfree, Lagrangian and particle nature. In this paper, some recent developments of the SPH in modelling explosion and impact problems will be introduced. A modified scheme for approximating kernel gradient (kernel gradient correction, or KGC) has been used in the SPH simulation to achieve better accuracy and stability. The modified SPH method is used to simulate a number of problems including 1D TNT detonation, linear shaped charge and explosively driven welding. The effectiveness of the modified SPH method has been demonstrated by comparative studies of the SPH results with data from other resources.

1 INTRODUCTION

Explosion and impact problems are generally characterized by the presence of shock waves, intense localized materials response and intensive loadings. A typical high explosive (HE) explosion consists of the detonation process through the HE and the later expansion process of the gaseous products to the surrounding medium[1]. For impact problems with different impacting speeds, solids under extreme situations behave like fluids. Two typical situations are high velocity impact (HVI) and penetration. In HVI, the kinetic energy of the system dominates and forces the solid material to deform extremely and the material actually "flows". In the events of penetration, the materials can even be broken into pieces that "fly", in addition to the extremely large deformation[2].

Recently more and more analyses of explosion and impact problems are based on numerical simulations with the advancement of the computer hardware and computational techniques. Most of the applications are generally grid-based numerical methods such as the finite element methods (FEM) or finite difference methods (FDM). Though many successful achievements have been made for these methods in modeling explosion and impact problems, some numerical difficulties still exist. These numerical difficulties generally arise from large deformations, large inhomogeneities, and moving interfaces, movable boundaries when simulating explosion and impact problems including HE detonation and explosion of explosive gas, deformation of solid materials, and interaction of fluids and solids.

Smoothed particle hydrodynamics (SPH) method[3, 4] is a Lagrangian, meshfree particle method. In SPH, particles are used to represent the state of a system and these particles can freely move according to internal particle interactions and external forces. Therefore it can naturally obtain history of fluid/solid motion, and can easily track material deformations, free surfaces and moving interfaces. During the last decades, SPH has been applied to modeling high explosive detonation and explosion, and hydrodynamics with material strength such as impact and penetrations[1, 5-8]. However, most of the existing works on SPH modeling of explosion and impact problems are based on conventional SPH method, which is believed to have poor performances especially in modeling problems with highly disordered particles[5]. They usually lack quantitative and even qualitative comparisons with experimental results, and also lack validation and verification in energy conservation.

In this paper, we shall present a modified SPH model and the modified SPH model will be applied to a number of explosion and impact problems to demonstrate its effectiveness.

2 SPH METHODOLOGY

2.1 Basic concepts of SPH

In conventional SPH method, there are basically two steps in obtaining an SPH formulation, kernel and particle approximations. The kernel approximation is to represent a function and its derivatives in continuous form as integral representation using the smoothing function and its derivatives. In the particle approximation, the computational domain is discretized with a set of particles. A field function and its derivative can then be written in the following forms[9]

$$< f(\boldsymbol{x}_i) >= \sum_{j=1}^{N} \frac{m_j}{\rho_j} f(\boldsymbol{x}_j) W(\boldsymbol{x}_i - \boldsymbol{x}_j, h) \tag{1}$$

$$< \nabla f(\boldsymbol{x}_i) >= \sum_{j=1}^{N} \frac{m_j}{\rho_j} f(\boldsymbol{x}_j) \nabla_i W_{ij} \tag{2}$$

where $< f(\boldsymbol{x}_i) >$ is the approximated value of particle *i*; $f(\boldsymbol{x}_j)$ is the value of $f(\boldsymbol{x})$ associated with particle *j*; $\boldsymbol{x}_i$ and $\boldsymbol{x}_j$ are the positions of corresponding particles; m and ρ denote mass and density respectively; *h* is the smooth length; *N* is the number of the particles in the support domain; *W* is the smoothing function representing a weighted contribution of particle *j* to particle *i*. The smoothing function should satisfy some basic requirements, such as normalization condition, compact supportness, and Delta function behavior[4, 10].

2.2 SPH equations of motion

For hydrodynamics of fluids and solids with material strength, the following governing equations of continuum mechanics apply

$$\begin{cases} \dfrac{D\rho}{Dt} = -\rho \dfrac{\partial \boldsymbol{v}^{\beta}}{\partial \boldsymbol{x}^{\beta}} \\ \dfrac{D\boldsymbol{v}^{\alpha}}{Dt} = \dfrac{1}{\rho} \dfrac{\partial \sigma^{\alpha\beta}}{\partial \boldsymbol{x}^{\beta}} \\ \dfrac{De}{Dt} = \dfrac{\sigma^{\alpha\beta}}{\rho} \dfrac{\partial \boldsymbol{v}^{\alpha}}{\partial \boldsymbol{x}^{\beta}} \\ \dfrac{D\boldsymbol{x}^{\alpha}}{Dt} = \boldsymbol{v}^{\alpha} \end{cases} \tag{3}$$

where the scalar density ρ, and internal energy e, the velocity component $\boldsymbol{v}^{\alpha}$, and the total stress tensor $\sigma^{\alpha\beta}$ are the dependent variables. The spatial coordinates $\boldsymbol{x}^{\alpha}$ and time *t* are the independent variables. The summation in equation (3) is taken over repeated indices, while the total time derivatives are taken in the moving Lagrangian frame. The total stress tensor $\sigma^{\alpha\beta}$ in equation (3) is made up of two parts, one part of isotropic pressure *p* and the other part of shear stress $S^{\alpha\beta}$. The hydrodynamic pressure is computed from an equation of state (EOS). For explosive gas, as the isotropic pressure is much larger than components of viscous shear stress, the viscous shear stress can be neglected. For solid materials, the shear stress can be computed from the constitutive equations of corresponding materials. Therefore using above-mentioned SPH approximations, the following SPH equations of motion can be obtained

$$\begin{cases} \dfrac{d\rho_i}{dt} = \rho_i \sum\limits_{j=1}^{N} \dfrac{m_j}{\rho_i} \left(\boldsymbol{v}_i^{\beta} - \boldsymbol{v}_j^{\beta} \right) \dfrac{\partial W_{ij}}{\partial x_i^{\beta}} \\ \dfrac{d\boldsymbol{v}_i^{\alpha}}{dt} = -\sum\limits_{j=1}^{N} m_j \left(\dfrac{\sigma_i^{\alpha\beta}}{\rho_i^2} + \dfrac{\sigma_j^{\alpha\beta}}{\rho_j^2} + \Pi_{ij} \right) \dfrac{\partial W_{ij}}{\partial x_i^{\beta}} \\ \dfrac{de_i}{dt} = \dfrac{1}{2} \sum\limits_{j=1}^{N} m_j \left(\dfrac{P_i}{\rho_i^2} + \dfrac{P_j}{\rho_j^2} + \Pi_{ij} \right) \left(\boldsymbol{v}_i^{\beta} - \boldsymbol{v}_j^{\beta} \right) \dfrac{\partial W_{ij}}{\partial x_i^{\beta}} + \dfrac{1}{\rho_i} S_i^{\alpha\beta} \varepsilon_i^{\alpha\beta} + H_i \\ \dfrac{d\boldsymbol{x}_i^{\alpha}}{dt} = \boldsymbol{v}_i^{\alpha} \end{cases} \tag{4}$$

where $\varepsilon^{\alpha\beta}$ is the strain rate tensor, Π and H stand for the component of the deviator stress tensor, the artificial viscosity and the artificial heat separately[4].

2.3 Kernel gradient correction

The conventional SPH method has been hindered with low accuracy and its accuracy is also closely related to the distribution of particles, selection of smoothing function and the support domain. Though different approaches have been proposed to improve the particle inconsistency and hence the SPH approximation accuracy, these approaches are usually not preferred for hydrodynamic simulations because the reconstructed smoothing function can be partially negative, non-symmetric, and not monotonically decreasing.

Explosion and impact involve fast expansion of explosive gas, rapid deformation and even liquefaction of solid materials, and quick damage on target materials. These lead to highly disordered particle distribution, which can seriously influence computational accuracy of SPH approximations. Hence an SPH approximation scheme, which is of higher order accuracy and is insensitive to disordered particle distribution, is necessary.

In this paper, the kernel gradient in SPH approximations is improved with a kernel gradient correction (KGC) technique[11]. In the KGC technique, a modified or corrected kernel gradient is obtained by multiplying the original kernel gradient with a local reversible matrix $L(\boldsymbol{r}_i)$, which is obtained from Taylor series expansion method. In two-dimensional spaces, the new kernel gradient of the smoothing function $\nabla_i^C W_{ij}$ can be obtained as follows

$$\nabla_i^C W_{ij} = L(\mathbf{r}_i)\nabla_i W_{ij} \tag{5}$$

$$L(\mathbf{r}_i) = \left(\sum_j \begin{pmatrix} x_{ji}\dfrac{\partial W_{ij}}{\partial x_i} & y_{ji}\dfrac{\partial W_{ij}}{\partial x_i} \\ x_{ji}\dfrac{\partial W_{ij}}{\partial y_i} & y_{ji}\dfrac{\partial W_{ij}}{\partial y_i} \end{pmatrix} V_j \right)^{-1} \tag{6}$$

where $x_{ji} = x_j - x_i$, $y_{ji} = y_j - y_i$. It is found that for general cases with irregular particle distribution, variable smoothing length, and/or truncated boundary areas, the SPH particle approximation scheme with kernel gradient correction is of second order accuracy.

3 EUATIONS OF STATE AND CONSTITUTIVE MODELING

3.1 Equations of state

For ideal explosive, the standard Jones-Wilkins-Lee (JWL) equation[12] of state can be employed. The pressure of the explosive gas is given by

$$p = A(1-\frac{\omega\eta}{R_1})e^{-\frac{R_1}{\eta}} + B(1-\frac{\omega\eta}{R_2})e^{-\frac{R_2}{\eta}} + \omega\eta\rho_0 E \tag{7}$$

where η is the ratio of the density of the explosive gas to the initial density of the original explosive. e is the internal energy of the high explosive per unit mass. A, B, R_1, R_2 and ω are coefficients obtained by fitting the experimental data. E is the initial internal energy of the high explosive per unit mass. Values of the corresponding coefficients can be found in[4].

For non-ideal explosive, the JWL++ equation of state can be used to model the detonation process with chemical reaction, while the pressure consists of the contributions from both reacted and non-reacted explosive.

The Tillotson equation[13] is employed to describe pressure-volume-energy behavior of metals under high temperature, pressure and strain rate as follows

$$
\begin{aligned}
p_1 &= (a+\frac{b}{\omega_0})\ \eta\rho_0 e + A\mu + B\mu^2 \\
p_2 &= (a+\frac{b}{\omega_0})\ \eta\rho_0 e + A\mu \\
p_3 &= p_2 + \frac{(p_4 - p_2)(e-e_s)}{(e_s' - e_s)} \\
p_4 &= a\eta\rho_0 e + (\frac{b\eta\rho_0 e}{\omega_0} + A\mu e^{\beta x})e^{-\alpha x^2} \\
\eta &= \frac{\rho}{\rho_0}, \mu = \eta - 1, \omega_0 = 1 + \frac{e}{e_0\eta^2}
\end{aligned}
\tag{8}
$$

where a, b, A, B, α, β, e_0, e_s and e' are the parameters determined by the material, and p_1 to p_4 are the pressure of four different phases of material, e.g., the solid phase, the liquid phase, the vapor and liquid mixture, and the vapor phase[13].

3.2 Constitutive modeling

Johnson-Cook model[14] is one of the most popular constitutive models for numerical simulations of impact and penetration, and which are usually associated with high strain rate. The model considers the effects of the stress hardening, strain rate and the temperature evolution. The yield stress in Johnson-Cook model can be written as

$$\sigma_y = (A + B\varepsilon^{p^n})(1 + C\ln\dot{\varepsilon}^*)(1 - T^{*m}) \tag{9}$$

$$T^* = \frac{T - T_{room}}{T_{melt} - T_{room}} \tag{10}$$

where ε^p is the effective plastic strain, $\dot{\varepsilon}^*$ is a dimensionless strain rate, and T is the temperature. A, B, C, n and m are five parameters in the Johnson-Cook model that need to be determined from the torsion test under different strain rate, the Hopkinson Pressure Bar test with different temperatures, and the Standard static tensile test. Detailed parameters in the Johnson-Cook model for different metals can be found in[15].

4 APPLICATIONS

4.1 Detonation of a 1D ANFO bar

The conventional SPH method was successfully applied to model the detonation of a 1D TNT bar, which is of idea explosive with a constant detonation speed[1]. In this work, the detonation process of a 1D ANFO bar is modeled using the method, while ANFO is a non-ideal explosive and the JWL++ equation of state is used. The ANFO bar is 0.1m long, and

4000 particles are used in the simulation. Figure 1 shows the pressure profiles along the slab at 1 μs interval from 1 to 14 μs. The von Neumann spike can be well captured in this simulation by using the JWL++ equation of state. Instead, if using the conventional JWL equation of state, it is only feasible to reach steady C-J pressure. The entire detonation process costs 17 μs and the resultant detonation speed is 5725 m/s, which is very close to the experimental value of 5900 m/s.

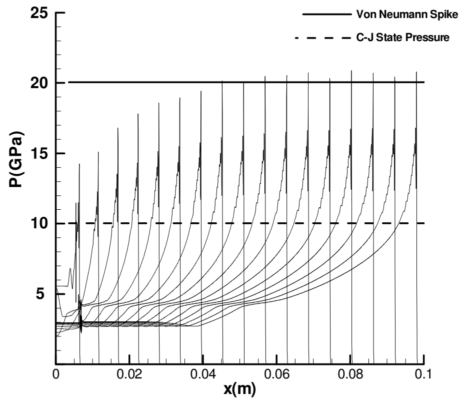

Figure 1: Pressure profiles along the 1D ANFO slab during the detonation process.

4.2 Linear shaped charge

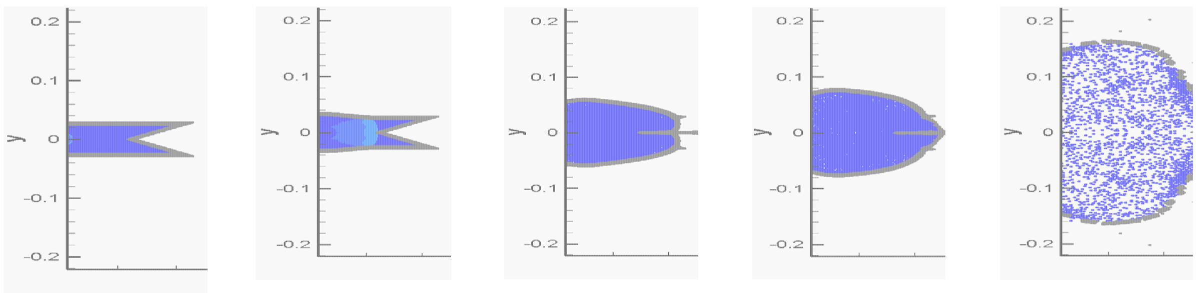

Figure 2: Shaped charge jet formation and penetration of a target plate.

Shaped charge is a frequently used form of explosive charge for military and industrial applications. It can produce powerful metal jet and lead to stronger penetration effects onto targets than normal charges. After the explosion of high explosive, the detonation produced explosive gas can exert tremendous pressure on surrounding metal case and liner with very large deformation and even quick phase-transition. There are some preliminary works of using SPH to model shaped charges. For example, Liu et al. first simulated the detonation and explosion process of two-dimensional shaped charges with different shapes of cavity [16] using SPH method. It is found that SPH can effectively model the explosive gas jet formation and dispersion. That work did not consider surrounding metal case and liner which present additional challenges in numerical simulation due to the existence of multi-material (explosive-metal) and multi-phase (solid-gas-liquid).

In this work, a practical linear shaped charge is modeled with metal case and linear. Figure 2 shows the entire process of HE detonation and explosion, explosion-driven metal deformation and jet formation as well the penetrating of a target aluminum plate at typical instants. The obtained snapshots with metal jet and penetrating effects agree well with experimental observations.

4.2 Explosive welding

Explosive welding can be used to bond two dissimilar metal plates to obtain a metal composite with better performance. It is attractive when conventional fusion welding techniques are not able to combine two metal plates together.

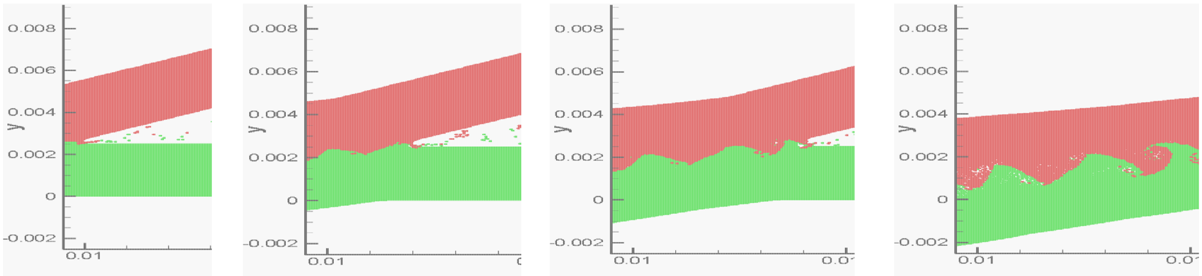

Figure 3: SPH modeling of the explosive welding process.

Figure 3 shows the SPH simulation of explosive (TNT) welding of two steel plates. It is observed that with the detonation and explosion of TNT, the high pressure explosive gas drives the flyer plate, which impacts on and interacts with the substrate (base plate). As the detonation wave moves rightwards, the interaction of the flyer plate and substrate also move rightwards. A metal jet produces between the two steel plates and a wave-like welding pattern can be obtained during the explosive welding process, which is also observable in laboratory experiments. Figure 4 shows a full scale view of the wave-like welding pattern during the explosive welding process.

Figure 4: A full scale view of the wave-like welding pattern.

12 CONCLUSIONS

- A modified SPH method with kernel gradient correction is developed for modeling explosion and impact problems.
- The modified SPH method has been applied to a number of applications including the detonation of a 1D ANFO bar, linear shaped charge jet formation and penetration effects on a target plate, and the explosive welding of two steel plates. The inherent physics can be well captured and the obtained numerical results are agreeable with experimental observations.

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